Mass Extrapolations in the Region of Deformed Rare Earth Nuclei

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Abstract. A procedure based on the regularity property of the mass surface is proposed to make predictions for the masses of neutron rich deformed nuclei in the rare earth region. Tables are given for the estimated masses; they extend up to the presumed limit of the deformation region.

A striking aspect of the chart of nuclides in the rare earth region is the deep "gulf" present on the neutron rich side. In this "gulf" the last known isotopes are often only 4−6 neutrons away from the stable isotopes while model predictions for the neutron drip line lay much further away. The limit of nuclei for which masses are known comes even closer. The onset of important deformation effects starting above N=88−90 is another characteristic of the region. In addition, some isotopic chains contain a rather small number of measured masses, centered around the stable isotopes. All these facts make this region a difficult one for mass predictions. Indeed, a comparison between the values given by various models for the isotopes beyond the last measured one indicates a growing divergence when one goes away from the last experimental values.

Based on the global property of regularity of the surface of masses, a method has been developed [1] to extrapolate starting from the known masses into the adjacent regions. It starts from the observation that derivative quantities like S_{2n} or S_{2p} (which are not affected by the staggering effects due to pairing) align themselves on straight lines when displayed as a function of neutron or respectively proton number. That will suggest a quadratic dependence on N or Z. This is valid only for a regular region in which neither shell (or subshell) closure appears, nor deformations in the ground state. At a closer look, these lines show a slight curvature. Consequently we tried a cubic (in N and Z) local fit of the masses of nuclei comprised in between two magic numbers both for neutrons and protons. Perhaps the most convenient region to test such a procedure is that of nuclei having N and Z in between magic numbers 28 and 50, as can be seen in [2]. Indeed, the result was quite encouraging: the *rms* deviation of the fitted values with respect to the data was 67 keV, while the same *rms* was higher for other model predictions [3]; e.g. 106

keV for the model of Duflo with 12 parameters [4], or 161 keV for the macroscopic-microscopic model of Möller [5]. In principle, the described method could provide reliable extrapolations for the next 4−5 masses, but in some particular cases its range of validity may extend further away. The procedure has been tested simply by excluding from the fit few (3−4) of the last known masses in each isotopic chain; the retrieved values agreed excellently with the real ones. For nuclei in the rare earth region the method encounters serious difficulties because here the regularity property is broken by the extra binding brought by the onset of deformation. However, one can still apply it to the region of masses with $50 \leq Z \leq 82$ and $82 \leq N \leq 126$ from which the deformed nuclei have been excluded. Though the number of nuclei left after this procedure is rather small, the fit is stable and leads to a hypothetical smooth mass surface for which the deformations are absent. By comparing to the real mass surface, the deformation region shows up prominently, presenting neat contours and a well developed symmetry The deformation sets in after N=88 and its amplitude grows gradually up to a maximum value; then it starts decreasing and disappears at N=116. The extension on Z ranges from Cs to Ir (with a small effect in both cases), having a maximum amplitude around Z=68. The position of maximum overbinding due to deformation along each isotopic chain varies from N=100 for small Z to N=106 for large Z. While for high Z the isotopic chains are almost complete from the point of view of measured masses, for lower Z the chains become ever shorter. Upper chains may therefore provide information on the trends that can be used to complete the others. This operation is facilitated by the continuous comparison with the hypothetical undeformed mass surface where all isotopic chains should land at the end of the deformation region. The amplitude of the overbinding brought by the deformation could also be estimated and amounts to 5 MeV at the maximum of this effect (for ^{168}Dy). Interestingly, most of the systematic values given in the tables of Audi and Wapstra [6], lay very close or overlap the extrapolated values. Table 1 is a list of masses estimated by this procedure for nuclei supposed to belong to the deformation region and that are not yet measured, from Xe to Ta. Only the values placed after the systematic values of Audi and Wapstra are given.

New mass measurement in this region, the sole criterium of validity for extrapolations and mass models are therefore strongly advocated.

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TABLE 1. Mass excess predictions (in MeV) for the rare earth deformed nuclei situated between Xe and Ta. The predictions start after the last systematic value in the tables of Audi and Wapstra and extend for each isotopic chain up to the expected end of the deformation region.

Nucleus	Mass excess (MeV)	Nucleus	Mass excess (MeV)	Nucleus	Mass excess (MeV)	Nucleus	Mass excess (MeV)
148Xe	–40.130	170Cs	84.240	174La	76.480	166Nd	–21.150
149Xe	–34.580	171Cs	91.290		.	167Nd	–15.270
150Xe	–30.630	172Cs	100.010	158Ce	–36.940	168Nd	–10.770
151Xe	–24.690		.	159Ce	–31.640	169Nd	–4.550
152Xe	–20.400	154Ba	–33.840	160Ce	–27.820	170Nd	0.300
153Xe	–14.070	155Ba	–28.240	161Ce	–22.120	171Nd	6.810
154Xe	–9.400	156Ba	–24.180	162Ce	–17.980	172Nd	12.010
155Xe	–2.720	157Ba	–18.190	163Ce	–11.910	173Nd	18.780
156Xe	2.300	158Ba	–13.810	164Ce	–7.330	174Nd	24.350
157Xe	9.310	159Ba	–7.480	165Ce	–0.920	175Nd	31.440
158Xe	14.680	160Ba	–2.720	166Ce	4.020	176Nd	37.380
159Xe	22.040	161Ba	3.960	167Ce	10.800	177Nd	44.700
160Xe	27.750	162Ba	9.110	168Ce	16.170		.
161Xe	35.430	163Ba	16.150	169Ce	23.310	164Pm	–38.470
162Xe	41.480	164Ba	21.670	170Ce	29.050	165Pm	–34.930
163Xe	49.530	165Ba	29.020	171Ce	36.470	166Pm	–30.030
164Xe	55.880	166Ba	34.890	172Ce	42.580	167Pm	–26.030
165Xe	64.220	167Ba	42.570	173Ce	50.260	168Pm	–20.710
166Xe	70.730	168Ba	48.750	174Ce	56.710	169Pm	–16.240
167Xe	79.380	169Ba	56.760	175Ce	64.660	170Pm	–10.560
168Xe	86.210	170Ba	63.270		.	171Pm	–5.640
169Xe	95.200	171Ba	71.560	160Pr	–36.440	172Pm	0.380
170Xe	102.360	172Ba	78.390	161Pr	–32.730	173Pm	5.610
171Xe	111.660	173Ba	87.000	162Pr	–27.590	174Pm	11.900
	.		.	163Pr	–23.480	175Pm	17.390
152Cs	–30.230	156La	–33.630	164Pr	–17.960	176Pm	24.000
153Cs	–26.040	157La	–29.690	165Pr	–13.430	177Pm	29.810
154Cs	–20.330	158La	–24.240	166Pr	–7.510	178Pm	36.570
155Cs	–15.750	159La	–19.950	167Pr	–2.600		.
156Cs	–9.650	160La	–14.140	168Pr	3.700	166Sm	–40.660
157Cs	–4.710	161La	–9.440	169Pr	9.010	167Sm	–35.900
158Cs	1.720	162La	–3.310	170Pr	15.640	168Sm	–32.350
159Cs	7.000	163La	1.760	171Pr	21.360	169Sm	–27.190
160Cs	13.800	164La	8.260	172Pr	28.250	170Sm	–23.260
161Cs	19.430	165La	13.730	173Pr	34.310	171Sm	–17.710
162Cs	26.510	166La	20.540	174Pr	41.490	172Sm	–13.360
163Cs	32.500	167La	26.380	175Pr	47.960	173Sm	–7.500
164Cs	39.940	168La	33.520	176Pr	55.360	174Sm	–2.780
165Cs	46.300	169La	39.750		.	175Sm	3.370
166Cs	54.060	170La	47.210	162Nd	–39.370	176Sm	8.480
167Cs	60.770	171La	53.750	163Nd	–34.320	177Sm	14.920
168Cs	68.840	172La	61.510	164Nd	–30.710	178Sm	20.410
169Cs	75.850	173La	68.430	165Nd	–25.220	179Sm	27.060

TABLE 1. (continuation)

Nucleus	Mass excess (MeV)	Nucleus	Mass excess (MeV)	Nucleus	Mass excess (MeV)	Nucleus	Mass excess (MeV)
168Eu	–39.350	180Gd	5.700	183Dy	–1.290	183Tm	–27.900
169Eu	–35.920	181Gd	11.750		.	184Tm	–23.760
170Eu	–31.140		.	176Ho	–38.930	185Tm	–19.960
171Eu	–27.330	172Tb	–39.260	177Ho	–35.770	186Tm	–15.650
172Eu	–22.200	173Tb	–35.960	178Ho	–31.630		.
173Eu	–18.000	174Tb	–31.440	179Ho	–28.120	182Yb	–38.580
174Eu	–12.530	175Tb	–27.760	180Ho	–23.690	183Yb	–34.830
175Eu	–7.990	176Tb	–22.990	181Ho	–19.880	184Yb	–32.070
176Eu	–2.250	177Tb	–19.020	182Ho	–15.080	185Yb	–28.000
177Eu	2.650	178Tb	–13.970	183Ho	–10.930	186Yb	–24.880
178Eu	8.630	179Tb	–9.620	184Ho	–5.990	187Yb	–20.630
179Eu	13.970	180Tb	–4.300		.		.
180Eu	20.110	181Tb	0.400	178Er	–40.020	185Lu	–33.580
	.	182Tb	5.920	179Er	–35.940	186Lu	–29.960
170Gd	–40.890		.	180Er	–33.100	187Lu	–26.810
171Gd	–36.310	174Dy	–40.590	181Er	–28.710	188Lu	–23.090
172Gd	–32.910	175Dy	–36.310	182Er	–25.430		.
173Gd	–28.000	176Dy	–33.110	183Er	–20.690	187Hf	–32.950
174Gd	–24.260	177Dy	–28.530	184Er	–16.940	188Hf	–30.640
175Gd	–18.990	178Dy	–24.940	185Er	–12.100	189Hf	–26.970
176Gd	–14.900	179Dy	–20.080		.	190Hf	–25.140
177Gd	–9.330	180Dy	–16.180	180Tm	–38.010		.
178Gd	–4.910	181Dy	–11.040	181Tm	–35.070	189Ta	–31.560
179Gd	0.960	182Dy	–6.730	182Tm	–31.210	190Ta	–28.360